The SIP Convention for Representing Distortion in FITS Image Headers

submitted by: David L. Shupe, IPAC / Caltech
Richard N. Hook, ST-ECF / ESO

1   Introduction

The Simple Imaging Polynomial, or SIP, convention provides a straightforward means for storing distortion information in FITS image headers. SIP was initially developed before the launch of the *Spitzer Space Telescope*. Images from the *Spitzer* instruments are distorted by a few percent relative to a regular sky grid. This distortion, expressed as a function of pixel position, is well-represented by polynomials, and it was desired to store the distortion information in the FITS headers of each Basic Calibrated Data (BCD) product. Writing the coefficients for each image was motivated particularly by the optics of the Multiband Imaging Photometer for Spitzer (MIPS) instrument (Rieke et al. 2004)—the distortion changes with scan mirror position, and hence from one image to the next.

The development of the SIP convention proceeded in parallel with work on the World Coordinate System (WCS) FITS standard. The first two papers in this series (Greisen & Calabretta 2002, "Paper I"; and Calabretta & Greisen 2002, "Paper II") specifying the WCS keywords (sans distortion) have been approved by the IAU FITS Working Group and are now standard. "Paper IV" addressing distortion has been drafted (http://www.atnf.csiro.au/people/mcalabre/WCS/index.html) but is not yet final. The SIP keywords are compliant with the first two papers, and have been influenced by early discussions of Paper IV, but are distinct from the proposed keywords in Paper IV.

This document is an expanded version of a paper presented at the 2004 ADASS conference (Shupe et al. 2005). The authors of that paper include the main contributors to the formulation and initial implementation of the SIP convention. The derivation of distortion coefficients for the Spitzer MIPS instrument using this convention are described in Morrison et al. 2007.

2   Definitions of the Distortion Keywords

The SIP convention derives its name from the four characters '-SIP' that are appended to the values of CTYPE1 and CTYPE2. These extra characters were included in early drafts of Paper IV to denote the distortion representation; however, later drafts dropped this form. We chose '-SIP' to be distinct from the '-PLP' that was to be used in Paper IV for polynomials, and because it has the useful mnemonic "Simple Imaging Polynomial". Note that this naming convention pre-dates the 3.0 version of the FITS Standard and is inconsistent with the restrictions on the form of the CTYPE*i* keyword values as specified in Section 8.2 of that document.

We define u, v as relative pixel coordinates with origin at CRPIX1, CRPIX2. Following Paper II, x, y are "intermediate world coordinates" in degrees, with origin at CRVAL1, CRVAL2. Let $f(u, v)$

and $g(u, v)$ be the quadratic and higher-order terms of the distortion polynomial. Then

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \mathtt{CD1\_1} & \mathtt{CD1\_2} \\ \mathtt{CD2\_1} & \mathtt{CD2\_2} \end{pmatrix} \begin{pmatrix} u + f(u, v) \\ v + g(u, v) \end{pmatrix} \tag{1}$$

We define $\mathtt{A\_p\_q}$ and $\mathtt{B\_p\_q}$ as the polynomial coefficients for polynomial terms $u^p v^q$. Then

$$f(u, v) = \sum_{p,q} \mathtt{A\_p\_q} u^p v^q, \quad p + q \leq \mathtt{A\_ORDER}, \tag{2}$$

$$g(u, v) = \sum_{p,q} \mathtt{B\_p\_q} u^p v^q, \quad p + q \leq \mathtt{B\_ORDER}. \tag{3}$$

For example, for a third-order polynomial,

$$f(u, v) = \mathtt{A\_2\_0} u^2 + \mathtt{A\_0\_2} v^2 + \mathtt{A\_1\_1} uv + \mathtt{A\_2\_1} u^2 v + \mathtt{A\_1\_2} uv^2 + \mathtt{A\_3\_0} u^3 + \mathtt{A\_0\_3} v^3$$

$\mathtt{A\_ORDER}$ and $\mathtt{B\_ORDER}$ can take on integer values ranging from 2 to 9. The default value of any unspecified $\mathtt{A\_p\_q}$ and $\mathtt{B\_p\_q}$ polynomial coefficients is 0.0.

The $\mathtt{CD}i\_j$ keywords encode skew as well as rotation and scaling. The CD-matrix values together with the higher-order distortion polynomials, as in Equations 1, 2, and 3, define a unique transformation from pixel coordinates to the plane-of-projection.

For Spitzer, we also provide polynomials for the reverse transformation, for fast inversion. Corrected pixel coordinates U, V are found from

$$\begin{pmatrix} U \\ V \end{pmatrix} = CD^{-1} \begin{pmatrix} x \\ y \end{pmatrix} \tag{4}$$

then the original pixel coordinates are computed by

$$u = U + F(U, V) = U + \sum_{p,q} \mathtt{AP\_p\_q} U^p V^q, \quad p + q \leq \mathtt{AP\_ORDER}, \tag{5}$$

$$v = V + G(U, V) = V + \sum_{p,q} \mathtt{BP\_p\_q} U^p V^q, \quad p + q \leq \mathtt{BP\_ORDER}. \tag{6}$$

To make a reasonably accurate reverse transformation, in general it is necessary to include linear terms in the reverse coefficients.

Finally, borrowing another idea from a Paper IV draft, the values of the keywords $\mathtt{A\_DMAX}$ and $\mathtt{B\_DMAX}$ give bounds on the maximum distortion over the array. These optional keywords could be used to estimate the maximum error that would result from not evaluating the distortion polynomial.

3   Example: *Spitzer*-IRAC Channel 4

We take as an example the distortion of the *Spitzer* Infrared Array Camera (IRAC) instrument (Fazio et al. 2004), which is characterized by cubic coefficients. Polynomial distortion of this form (plus linear terms) was fit to *Spitzer* data from the Great Observatories Origins Deep Survey program (S. Casertano, private communication). The linear terms are folded into the $\mathtt{CD}i\_j$. An excerpt from an actual BCD header produced by the Spitzer pipeline for IRAC Channel 4 is shown below.

```
CTYPE1  = 'RA---TAN-SIP'      / RA---TAN with distortion in pixel space
CTYPE2  = 'DEC--TAN-SIP'      / DEC--TAN with distortion in pixel space
CRVAL1  =     202.581507417836 / [deg] RA at CRPIX1,CRPIX2 (using Pointing Recon
CRVAL2  =     47.2465528124827 / [deg] DEC at CRPIX1,CRPIX2 (using Pointing Reco
CRPIX1  =                 128. / Reference pixel along axis 1
CRPIX2  =                 128. / Reference pixel along axis 2
CD1_1   = 0.000248349650353678 / Corrected CD matrix element with Pointing Recon
CD1_2   = 0.000232107213140475 / Corrected CD matrix element with Pointing Recon
CD2_1   = 0.000232418393583541 / Corrected CD matrix element with Pointing Recon
CD2_2   = -0.000246562617306562 / Corrected CD matrix element with Pointing Reco
A_ORDER =                    3 / polynomial order, axis 1, detector to sky
A_0_2   =           9.0886E-06 / distortion coefficient
A_0_3   =           4.8066E-09 / distortion coefficient
A_1_1   =           4.8146E-05 / distortion coefficient
A_1_2   =          -1.7096E-07 / distortion coefficient
A_2_0   =             2.82E-05 / distortion coefficient
A_2_1   =           3.3336E-08 / distortion coefficient
A_3_0   =          -1.8684E-07 / distortion coefficient
A_DMAX  =                2.146 / [pixel] maximum correction
B_ORDER =                    3 / polynomial order, axis 2, detector to sky
B_0_2   =           4.1248E-05 / distortion coefficient
B_0_3   =          -1.9016E-07 / distortion coefficient
B_1_1   =           1.4761E-05 / distortion coefficient
B_1_2   =           2.1973E-08 / distortion coefficient
B_2_0   =          -6.4708E-06 / distortion coefficient
B_2_1   =          -1.8188E-07 / distortion coefficient
B_3_0   =           1.0084E-10 / distortion coefficient
B_DMAX  =                1.606 / [pixel] maximum correction
AP_ORDER=                    3 / polynomial order, axis 1, sky to detector
AP_0_1  =           3.6698E-06 / distortion coefficient
AP_0_2  =          -9.1825E-06 / distortion coefficient
AP_0_3  =          -3.8909E-09 / distortion coefficient
AP_1_0  =          -2.0239E-05 / distortion coefficient
AP_1_1  =          -4.8946E-05 / distortion coefficient
AP_1_2  =           1.7951E-07 / distortion coefficient
AP_2_0  =          -2.8622E-05 / distortion coefficient
AP_2_1  =          -2.9553E-08 / distortion coefficient
AP_3_0  =           1.9119E-07 / distortion coefficient
BP_ORDER=                    3 / polynomial order, axis 2, sky to detector
BP_0_1  =          -2.1339E-05 / distortion coefficient
BP_0_2  =           -4.189E-05 / distortion coefficient
BP_0_3  =           1.9696E-07 / distortion coefficient
BP_1_0  =           2.8502E-06 / distortion coefficient
BP_1_1  =          -1.5089E-05 / distortion coefficient
BP_1_2  =          -2.0219E-08 / distortion coefficient
BP_2_0  =           6.4625E-06 / distortion coefficient
```

```
BP_2_1   =              1.849E-07 / distortion coefficient
BP_3_0   =             -7.6669E-10 / distortion coefficient
```

In this case, the reverse coefficients have the opposite sign and roughly the same absolute values as the corresponding forward coefficients. However, this is not true for some more distorted fields of view, so the Spitzer headers retain the reverse coefficients in general.

The Spitzer Science Center has developed library routines to implement this coefficient naming scheme. The functions key off the extended `CTYPEn`. The order in which the keywords are displayed in the example is the order in which the software searches for them and is the most efficient for lookups using CFITSIO.

4 Software that Reads and Applies the Coefficients

The usefulness of this convention was greatly enhanced by the generous efforts of a number of individuals who added support to their software before the first release of *Spitzer* data in 2004. The mosaicking package MOPEX (Makovoz & Khan 2005) applies the SIP distortion coefficients in the Spitzer Science Center pipelines. Support has also been added to IPAC's Skyview display program (http://www.ipac.caltech.edu/Skyview/). Doug Mink implemented Spitzer distortion support in his WCS routines (http://tdc-www.harvard.edu/software/wcstools). SAOimage and DS9 use these routines and hence automatically handle the SIP distortions. The Montage software (http://montage.ipac.caltech.edu) (Laity et al. 2004) also uses Mink's routines and applies the coefficients. Support in the GAIA viewer has been added via David Berry's AST library. Wayne Landsman has added support to the IDL ASTROLIB. The Drizzle software (Fruchter & Hook 2002) has also been modified to read these coefficients.

Finally we note that the astrometry.net service also uses the SIP convention for encoding the non-linear parts of the distortions it calculates in arbitrary images.

5 SIP for Hubble

Of the cameras currently on board the Hubble Space Telescope, the distortion is largest by far for the Wide Field Channel (WFC) of the Advanced Camera for Surveys (ACS) where it amounts to more than fifty pixels at the corner of the image in addition to an even larger (linear) skew term. The newer Wide Field Camera 3, to be installed in October 2008, has similarly large distortions.

The image distortion for Hubble cameras is currently characterized by a FITS table known as the Image Distortion Correction Table (IDCTAB) that includes information about the scale and orientation of the instrument aperture in the telescope focal plane as well as the distortion polynomial coefficients. Software has been developed that will combine the IDCTAB information with the normal information from the telescope's pointing control software to write out a header which makes the header WCS keywords fully compatible with the table values and also populates the SIP-keywords (or at least the most important ones). An example of the resultant header is given in Table 1.

Keyword	Value	Keyword	Value
CTYPE1	'RA---TAN-SIP'	CTYPE2	'DEC--TAN-SIP'
CRPIX1	2048.0	CRPIX2	1024.0
CRVAL1	5.6260667398471	CRVAL2	-72.076963036772
CD1_1	-7.8481866550866E-06	CD2_1	1.1406694624771E-05
CD1_2	1.0939720432379E-05	CD2_2	8.6942510845452E-06
A_0_2	2.1634068532689E-06	B_0_2	-7.2299995118730E-06
A_1_1	-5.194753640575E-06	B_1_1	6.1778338717084E-06
A_2_0	8.543473309812E-06	B_2_0	-1.7442694174934E-06
A_0_3	1.0622437604068E-11	B_0_3	-4.2102920235938E-10
A_1_2	-5.2797808038221E-10	B_1_2	-6.7603466821178E-11
A_2_1	-4.4012735467525E-11	B_2_1	-5.1333879897858E-10
A_3_0	-4.7518233007536E-10	B_3_0	8.5722142612681E-11
A_0_4	1.4075878614807E-14	B_0_4	6.5531313110898E-16
A_1_3	-1.9317154005522E-14	B_1_3	1.3892905568706E-14
A_2_2	3.767898933666E-14	B_2_2	-2.9648166208490E-14
A_3_1	5.0860953083043E-15	B_3_1	-2.0749495718513E-15
A_4_0	2.5776347115304E-14	B_4_0	-1.812610418272E-14
A_ORDER	4	B_ORDER	4

Table 1: SIP coefficients for the Hubble ACS Wide Field Channel.

Currently the writing of these SIP keywords is an unsupported feature for Hubble data. However, it is planned to formally include such headers in future to provide users with a full, self-describing distortion model without the need for access to external files in non-standard formats. The software to read the coefficients and apply them to remove image distortion already exists within the standard Hubble data processing tools.

6 Issues and Caveats

The SIP convention has been in use for several years and is becoming more widespread. To gauge feelings about it we recently have asked for comments from several people who have used it for and are familiar with its features. We are grateful for their input and time. In this section we summarize some possible limitations of the standard.

The SIP specification provides for "reverse" coefficients to allow the mapping of sky coordinates to pixels to be performed rapidly, without the need for iterative inversion techniques. The Spitzer mosaicking tool MOPEX relies on the reverse coefficients for its default interpolation mode, as it maps output pixel corners back to the original distorted images. The reverse polynomial is, however, only an approximation and in general cannot be the exact inverse of the forward polynomial. As a result mapping a pixel to celestial coordinates and back does not yield back precisely the original coordinates. For example, in the IRAC channel 4 distortion listed above, mapping pixel coordinate (1.0,1.0) to the sky and back leads to a difference of about 0.014 pixels. It can be argued that such a difference is negligible for practical applications, and that distortions may not be measured to such accuracy anyway. Another drawback to the reverse coefficients is that they violate the principle of storing only the minimum information necessary in the FITS header – the forward coefficients

could be considered to contain all the necessary information.

A better approach, in hindsight, might have been to not include the "reverse" coefficients at all, but instead to invert the forward solution using an interative technique. For an arbitrary polynomial, it might not be possible to guarantee convergence of the inversion. For practical applications to distorted images, however, the size of the corrections are small and inversion will likely work well. It should be noted that the reverse coefficients for the examples given above are very nearly the same as the forward coefficients with the sign reversed. The starting points for any iterative inversion are well-determined and the solution should be reached rapidly.

Based on these considerations, use of the reverse coefficients should be considered optional although this may create problems for existing tools such as WCSTools which have already been coded to use the reverse coefficients.

Another area of concern concerns possible loss of accuracy in the calculations under some circumstances. In the case of large pixel coordinate values, and high order polynomials, the terms can grow large. In some cases it is necessary to take the differences between polynomial terms that are much larger than the final result — a classic case where accuracy can be lost. It clearly helps significantly to use double precision floating point numbers and hence around 15 significant figures of accuracy. We would strongly discourage the use of single precision but it remains a question for software developers and is not imposed by the convention itself. A sufficient number of significant digits should be used to specify the distortion coefficients in the FITS header. As shown in the examples above, the high-order coefficients become small and must be specified in scientific notation with large negative exponents.

An alternative solution to avoid loss of accuracy, or overflow or underflow problems, is to introduce a scaling term so that pixel values are scaled into the range -1 to $+1$. This could quite easily be done with by adding an optional keyword, that defaults to 1.0 in order not to invalidate existing headers.

Another comment we have received is that the form of the keywords is relatively simple, so much so that someone else might accidentally use one of the distortion keywords for another purpose elsewhere in the FITS header, thereby corrupting the distortion information.

7 Possible New Features

We have also asked for views about possible extensions to SIP.

One limitation of this current convention is that only regular polynomials are allowed – not Chebyshev or Lagrange for example. As a result higher-order polynomials can diverge at the edges of images where they are less well constrained and this could cause difficulties under some conditions. However, adding these would be significant work and we do not think it should be considered at present. If these were implemented, we would recommend a different three-letter suffix for `CTYPE1` and `CTYPE2`, or some other means to maintain a distinction from the simple polynomials currently used.

In the current convention the distortion origin is forced to be at (`CRPIX1, CRPIX2`). However, in many cases it is natural for the distortion center to lie at a different location. It has been suggested that additional keywords could be used to specify the distortion center, and if these are

not present then the default of (CRPIX1, CRPIX2) is used. Although the introduction of a different center may have advantages there are also significant drawbacks and some of the simplicity of the original scheme is lost. In particular, the CD matrix would no longer contain a correct description of pixel scales, skew and orientation at the point (CRPIX1, CRPIX2). We note that it is always possible to exactly shift the distortion origin to (CRPIX1, CRPIX2) with the result as a polynomial of the same order, although it is possible in extreme cases that this will result in much larger terms and possible consequent accuracy loss.

8   Concluding Remarks

The SIP convention has proved to be applicable to many imaging situations and its simplicity has made implementation and use easy. Many people feel it is the natural solution without excess detail. However, this simplicity naturally limits its generality and largely restricts the applicability of SIP to simple cameras, unlike the much more extensive general proposals in FITS Paper 4 that include multi-dimensional support that cover far more cases beyond just simple imaging.

9   Acknowledgments

10   References

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